First-excursion stochastic incremental dynamics methodology for hysteretic structural systems subject to seismic excitation

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Abstract
A novel efficient stochastic incremental dynamics methodology considering first-excursion probability for nonlinear structural systems subject to stochastic seismic excitations in alignment with contemporary aseismic codes provisions is developed. To this aim, an approximate nonlinear stochastic dynamics technique for conducting first-passage probability density function (PDF) based stochastic incremental dynamic analysis is developed. Firstly, an efficient stochastic iterative linearization methodology is devised achieving convergence of the equivalent system damping ratios with the damping premises of the excitation response spectrum leading to a coherent determination of a robust scalable intensity measure (IM) which bears direct relation to its damaging potential. Subsequently, utilizing the stochastically derived time-varying forced vibrational system properties in conjunction with a combination of deterministic and stochastic averaging treatment the first-excursion PDF is efficiently determined for each and every of the considered limit-state rules (LSs). Lastly, an incremental mechanization analogous to the one used in normal incremental dynamic analysis (IDA) is proposed to ensure the necessary compatibility for applications in the fields of structural and earthquake engineering. The back-and-forth twisting pattern of IDA curves which is related with the multiple points satisfaction of the very same limit-state rule encourages the study of the problem from a first-passage perspective considering timing in addition to the intensity of the excitation variable. The selected engineering demand parameter (EDP) of the first-excursion time constitutes an excellent response related variable with twofold meaning; it performs structural behavior monitoring considering intensity and timing information whereas it is inherently coupled with limit-state requirements. The developed stochastic dynamics technique provides with reliable higher order statistics (i.e., PDF) of the chosen EDP. A structural system comprising the bilinear hysteretic model serves as a numerical example for demonstrating the reliability of the proposed first-excursion PDF-based stochastic incremental dynamics methodology. Nonlinear response time-history analysis involving a large ensemble of Eurocode 8 spectrum compatible accelerograms is conducted to assess the accuracy of the proposed methodology in a Monte Carlo-based context.

Keywords: first-excursion probability density function, nonlinear stochastic dynamics, stochastic iterative linearization scheme, stochastic averaging, bilinear hysteretic systems, performance-based engineering

1 INTRODUCTION

In the engineering discipline of earthquake resistant structures nonlinearities naturally arise in various forms. This fact brings to the fore the need for a pertinent representation of the system model by considering thoroughly the underlying mechanisms which determine the system behaviour. In this setting, a suitable stochastic representation of seismic excitation in conjunction with nonlinear/hysteretic system modelling provides a solid basis for formulating a realistic analysis procedure (e.g. [1,2]). In general, a proper quantitative treatment of uncertainties is a fundamental prerequisite to derive reliable numerical predictions of the performance of engineering systems and structures. In this setting, the emerged concept of performance-based earthquake engineering (PBEE) supports the assessment of systems performance in a consistent and comprehensive manner by properly accounting for the ubiquity of uncertainties (e.g. [3-5]). Inherent in the philosophy of the PBEE is the definition of excitation related variables, known as intensity measures (IMs) (e.g., spectral acceleration, peak ground acceleration, etc), and of system response related variables known as engineering demand parameters (EDPs) (e.g., peak story drift, inter-story drift ratio, etc). Further, the information provided via the functional relationship between the IMs and the EDPs in conjunction with appropriately defined limit-state rules (LSs), is utilized for quantifying a chosen decision variable (e.g., life cycle-cost, financial loss, etc). Nevertheless, the determination of the above-mentioned functional relationship constitutes typically a computationally demanding and cumbrous task.

In the field of earthquake engineering, one of the commonly applied methodologies for estimating the functional relationship between the IMs and the EDPs is the incremental dynamic analysis (IDA) [6,7]. IDA aims at assessing the performance of structural systems subject to a suite of ground motion records, each scaled to several levels of seismic intensity; thus, performing a nonlinear response time-history analysis (RHA) for each and every scaled record. Note that each IDA curve is related to a specific ground motion record and each point of the curve corresponds to a specific ground motion intensity level and respective structural system response magnitude (e.g. peak story drift, inter-story drift ratio, etc); see Figs. 1a-b.

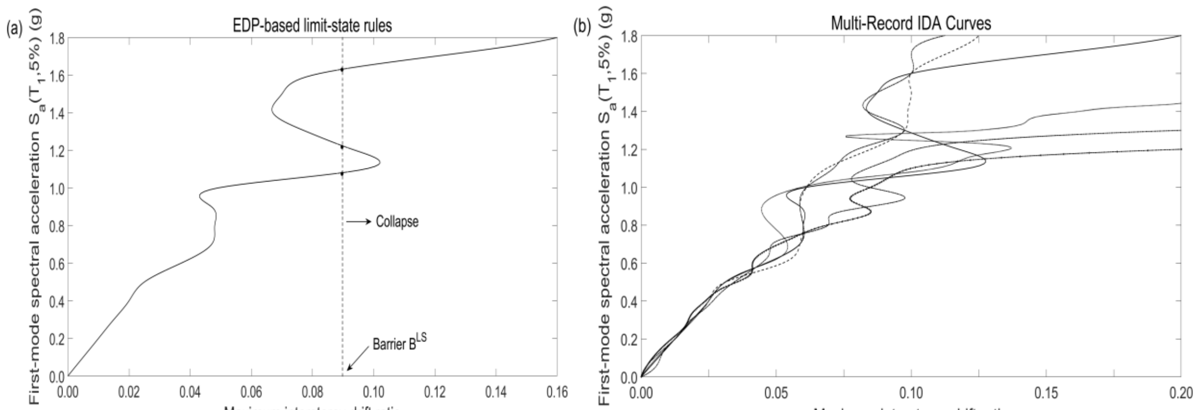

Figure 1. IDA curves of a T_1=1.1 s, 3-storey building structure (a) EDP-based rule for single-record IDA curve (b) Multi-record IDA curves for the 3-storey building structure subjected to seven different ground motion records.

A substantial debate has been raised regarding the potential bias in the IDA results derived from the selection of a limited number of scaled real recorded accelerograms. Note in passing, that the challenge of selecting and scaling earthquake records still remains a highly controversial issue in the relevant literature (e.g. [8,9]). Clearly, performing multi-record IDA within a probabilistic framework introduces considerable variability regarding the satisfaction of a limit-state rule which signals the onset of stiffness and strength degradation accompanied by the entrance of a structure in a specific limit/damage state. The back-and-forth twisting pattern which can be observed even in the case of a standard single-record IDA is related with the multiple points satisfaction of the very same limit-state rule (see Fig. 1a.). On the course of interpreting this fact, the admission of hardening issues which take the form of structural resurrection in the extreme case as well as phenomena of period elongation (e.g. [6]) can be found in the relevant literature. In practice, the twisting pattern is encouraged to be treated in a way that the portion above the first flatline could be discarded and only consider points up to this first sign of entrance in the damage state. The apparent choice of the lowest IM value which phenomenologically seems to be in alignment with the conservative engineering intuition, turns out to be of limited value given the induced bias in the selection of the seismic records. The counter-intuitive fact that a system performs lower responses when is excited by higher seismic intensities brings naturally to the fore the need for addressing also the timing parameter rather than just the intensity of the excitation variable; considering a scaled image of an accelerogram weak response cycles at the early part of the system response time-history can become strong enough to inflict yielding thus converting system behaviour for the upcoming stronger cycles. Clearly, the complexity of the mathematical entity of IDA curve is interwoven with scaling as well as timing ambiguity. In an attempt to address this twofold peculiarity, the herein study introduces the concept of first-excursion probability in conjunction with a novel IM involving response-based scaled excitation stochastic processes compatibly defined with contemporary aseismic code provisions; this modelling is designated towards addressing time and scaling ambiguity within a fully probabilistic framework.

This paper proposes an approximate nonlinear stochastic dynamics technique for conducting first-passage PDF-based stochastic IDA of nonlinear structural systems subject to stochastic seismic excitations defined in alignment with contemporary aseismic codes provisions (e.g. Eurocode 8) [10]. Firstly, an efficient stochastic iterative linearization scheme is devised achieving convergence of the equivalent damping ratios with the damping premises of the excitation response spectrum leading to reliable stochastically derived forced vibrational properties estimates which can track system nonlinear behavior. In this setting, a vector of carefully scaled stochastic processes with respect to the forced vibrational character of the system is induced. Subsequently, utilizing the stochastically derived forced vibrational system properties in conjunction with a combination of deterministic and stochastic averaging treatment the first-excursion PDF is efficiently determined for the considered limit-state rules. Lastly, an incremental mechanization analogous to the one used in the standard implementation of IDA regarding the scaling of intensity is employed to ensure the necessary compatibility.

The proposed first-excursion PDF-based stochastic incremental dynamics methodology differs, as compared with a typically applied IDA implementation, in the following aspects: (i) the ground motion is modeled in the form of a vector of stochastic processes rather than a suite of scaled earthquake records; this attribute partially addresses the issue of the induced bias in the process of records selection; (ii) a non-negative scalar scale factor would lead to scaled records which offer no information regarding their repercussions on a given structure [6]. In the herein study, an alternative scaling is employed which demonstrates enhanced relation to the excitation damaging potential by considering the effects of the induced nonlinearity in the determination of the scaled image of IM; (iii) the developed stochastic incremental dynamics analysis (SIDA) methodology determines higher order statistics of the selected EDP rather than simple estimates only of the mean and the standard deviation currently being the norm in the literature; (iv) the latter attribute enables the associated computational cost to be kept at a minimum level circumventing computationally demanding Monte Carlo Simulation (MCS);

(v) the back-and-forth twisting pattern of IDA curves which is related with the multiple points satisfaction of the very same limit-state rule dictates the need for studying the problem from a first-passage perspective rendering appropriate the selection of the limit-state first-excursion time as the EDP herein; (vi) it liberates the potential researcher from interpreting complex twisting patterns behaviour as conforming or non-conforming with a particular performance level.

In the remainder of this paper Sections 2.1-2.5 review the mathematical background supporting the developed framework, Section 2.6 furnishes pertinent comments on intriguing attributes and practical usage of the implementation technique, Section 3, presents an illustrative application of the framework to a yielding frame structure exposed to a properly defined vector of stochastic seismic excitations and assesses its accuracy against nonlinear response time-history analysis (RHA) data, and Section 4 summarizes the main conclusions.

2 MATHEMATICAL BACKGROUND

This section exemplifies the mathematical details involved in the development of the proposed efficient first-excursion PDF-based incremental stochastic dynamics methodology. Particular attention has been given on elucidating the various simplifications and assumptions made in light of numerical efficiency. To ensure the coherency in presenting the theoretical background material without sacrificing the readability of the manuscript a brief introduction including only the basic concepts associated with the generation of response spectrum compatible stochastic processes is given in the following subsection 2.1.

2.1 *Power spectra derivation for the response spectrum compatible stochastic processes*

The basic notion of the approach lies on fitting a stationary Gaussian acceleration process $\ddot{x}_g(t)$ of finite duration T_s, to an assigned elastic response uniform hazard spectrum (UHS) of specified modal damping ratio. In this context, the nonlinear equation which consists the basis for relating a damped pseudo-acceleration response spectrum $S_a(\omega_i, \zeta_j)$ to an one-sided power spectrum corresponding to a Gaussian stationary stochastic response process $r_i(t)$ in the frequency domain reads

$$S_a(\omega_i, \zeta_j) = \eta_{r_i} \omega_i^2 \sqrt{\lambda_{0,r_i}} \tag{1}$$

where η_{r_i} and λ_{0,r_i} stand for the peak factor and the variance of the response process $r_i(t)$ of an elastic oscillator of natural frequency ω_i and damping ratio ζ_j. The equation of motion for a viscously damped quiescent linear oscillator in the time domain reads

$$\ddot{r}_i(t) + 2\zeta_j \omega_i \dot{r}_i(t) + \omega_i^2 r_i(t) = -\ddot{x}_g(t). \tag{2}$$

where a dot over a variable signifies differentiation with respect to time t. Furthermore, the spectral moment of zeroth order of the stochastic process that appears in Eq.(1), reads for the general case of nth order

$$\lambda_{n,r_i} = \int_0^\infty \omega^n \frac{1}{(\omega_i^2 - \omega^2)^2 + \left(2\zeta_j \omega_i \omega\right)^2} G^{\zeta_j}(\omega) d\omega. \tag{3}$$

The determination of the peak factor η_{r_i} is related with the concept of first-passage problem. Next, assuming the hypothesis of a barrier outcrossing in clumps [11], the peak factor is expressed as

$$\eta_{r_i}(T_s, p) = \sqrt{2\ \ln\{2\ v_{r_i}[1 - \exp\left[-\delta_{r_i}^{1.2}\sqrt{\pi\ \ln(2\ v_{r_i})}\right]]\}} \tag{4}$$

where the spread factor δ_{r_i} and the mean zero crossing rate v_{r_i} of the stochastic response process $r_i(t)$ are defined as

$$\delta_{r_i} = \sqrt{1 - \frac{\lambda_{1,r_i}^2}{\lambda_{0,r_i}\,\lambda_{2,r_i}}} \quad \text{and} \quad v_{r_i} = \frac{T_s}{2\pi}\sqrt{\frac{\lambda_{2,r_i}}{\lambda_{0,r_i}}}(-\ln p)^{-1} \tag{5}$$

respectively. Note in passing that the evaluation of the stochastically compatible power spectrum $G^{\zeta_j}(\omega)$ characterizing in frequency domain a zero-mean input process of duration T_s, which does not appear explicitly in Eq.(1), requires a careful handling of the inverse stochastic dynamics problem. The peak factor η_{r_i} consists the critical factor by which the standard deviation of the considered elastic oscillator response should be multiplied to predict a level S_a below which the peak response will remain, with probability p introduced in Eq.(4). Based on Vanmarcke's [12] approximate formula for obtaining a reliable estimation of the variance of the response process $r_i(t)$ of an oscillator of natural frequency ω_i and damping ratio ζ_j, the following direct numerical scheme for the evaluation of the stochastically compatible power spectrum $G^{\zeta_j}(\omega)$ is derived

$$G^{\zeta_j}(\omega_i) = \begin{cases} \dfrac{4\zeta_o}{\omega_i\pi - 4\zeta_j\omega_{i-1}}\left(\dfrac{S_\alpha^2(\omega_i,\zeta_j)}{\eta_{r_i}^2} - \Delta\omega\displaystyle\sum_{k=1}^{i-1} G^{\zeta_j}(\omega_k)\right), & \omega_i > \omega_b^l \\ 0, & \omega_i \le \omega_b^l \end{cases} \tag{6}$$

where the discretization scheme $\omega_i = \omega_b^l + (i - 0.5)\Delta\omega$ is employed; see [13]. Clearly, a preselection of an input power spectrum shape has to be preceded for deriving a stochastically compatible spectrum (e.g. white-noise, Butterworth filtered Kanai-Tajimi, Clough-Penzien), according to the numerical scheme of Eq.(6). For a more detailed presentation as well as pertinent commentary on the topic the interested reader may resort to the works (e.g. [13,14]). The nonlinear stochastic dynamics technique unfolded in the following subsections is independent from the herein presented approach which only works as a necessary stepping stone allowing for the generation of power spectra characterizing the underlying code-compliant stochastic processes corresponding to the strong part of the imposed seismic excitation.

2.2 *Efficient nonlinear stochastic iterative linearization scheme*

2.2.1 Statistical linearization for nonlinear systems under stochastic seismic excitation

Many real engineering systems can be modelled adequately as single-degree-of-freedom (SDOF) systems (e.g. [15]). Consider a quiescent nonlinear SDOF oscillator base-excited by the response spectrum compatible acceleration stochastic process $\ddot{x}_g(t)$ whose dynamic behavior is governed by the differential equation,

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + g\big(x(t), \dot{x}(t)\big) = f(t) = -m\ddot{x}_g(t), \quad x(0) = \dot{x}(0) = 0 \quad (7)$$

where $g(x(t), \dot{x}(t))$ is the restoring force that can be either hysteretic or depend only on the instantaneous values of $x(t)$ and $\dot{x}(t)$; m, c and k stand for the oscillator mass, damping and stiffness respectively. In general, a filtered stationary stochastic process can be employed to represent a non-stationary stochastic process according to the concept proposed by Priestley [16] and later refined by Dalhaus introducing the class of locally stationary processes [17]. In this setting, the random force $f(t)$ can be expressed in the frequency domain via the power spectrum defined as $S_{ff}(\zeta_j, t, \omega) = |w(t)|^2 m^2 G^{\zeta_j}(\omega)$ where the time-variant filter $w(t)$ takes the form of a unit vector under the stationarity assumption. In the ensuing analysis, for the sake of efficiency an equivalent form of Eq.(7) is utilized

$$\ddot{x}(t) + 2\zeta_o\omega_o\dot{x}(t) + \omega_o^2 x(t) + g_o\big(x(t), \dot{x}(t)\big) = f_o(t) = -\ddot{x}_g(t), \qquad x(0) = \dot{x}(0) = 0 \quad (8)$$

where ζ_o and ω_o are the ratio of critical damping and the natural frequency corresponding to the linear oscillator (i.e., $g\big(x(t), \dot{x}(t)\big) = 0$); the restoring force $g_o\big(x(t), \dot{x}(t)\big) = g\big(x(t), \dot{x}(t)\big)/m$, $f_o(t) = f(t)/m$, $2\zeta_o\omega_o = c/m$ and $\omega_o^2 = k/m$. The Gaussian assumption which finds application in a number of studies in the earthquake engineering field (e.g. [2,14,15]) constitutes a stepping stone for the development of the herein proposed nonlinear stochastic dynamics methodology. Relying on the standard assumption that the response processes are Gaussian, and following statistical linearization (e.g. [15]), a linearized version of Eq. (8) is considered in the form

$$\ddot{x}(t) + 2\zeta_{eq}(t)\omega_{eq}(t)\dot{x}(t) + \omega_{eq}^2(t)x(t) = f_o(t) = -\ddot{x}_g(t), \quad x(0) = \dot{x}(0) = 0 \quad (9)$$

The equivalent system parameters, $\zeta_{eq}(t)$ and $\omega_{eq}(t)$ are obtained by minimizing the mean square of the error $\varepsilon(t)$ defined from the difference between Eqs.(8) and (9) as

$$\varepsilon(t) = \big(2\zeta_{eq}(t)\omega_{eq}(t) - 2\zeta_o\omega_o\big)\dot{x}(t) + (\omega_{eq}^2(t) - \omega_o^2)x(t) - g_o\big(x(t), \dot{x}(t)\big) \quad (10)$$

This criterion yields the following simplified expressions for the equivalent linear properties

$$\omega_{eq}^2(t) = \omega_o^2 + E\left[\frac{\partial g_o\big(x(t), \dot{x}(t)\big)}{\partial x(t)}\right] \text{ and } \zeta_{eq}(t) = \zeta_o \frac{\omega_o}{\omega_{eq}} + E\left[\frac{\partial g_o\big(x(t), \dot{x}(t)\big)}{\partial \dot{x}(t)}\right] \quad (11)$$

in which E[·] denotes the mathematical expectation operator. The response power spectrum can be found by working directly in the frequency domain as

$$S_{xx}(t, \omega) = |H(t, \omega)|^2 S_{f_o f_o}(\zeta_j, t, \omega) \quad (12)$$

where $H(t, \omega)$ stands for the complex frequency response function defined by considering the Fourier transform for both sides of the equation of motion (see Eq.(9))

$$H(t,\omega) = \frac{1}{\left[\left(\omega_{eq}^2(t) - \omega^2\right) + 2i\zeta_{eq}(t)\omega_{eq}(t)\omega\right]}. \quad (13)$$

Indeed, the spectral relationship of Eq. (12) is a straightforward generalization of the celebrated spectral relationship based on stationarity and on the Wiener–Khinchin theorem. Hence, the aforementioned approximation can be viewed as a quasi-stationary approach which can be quite accurate for a wide range of systems of engineering interest (e.g. [18,19]). Note that the spectral input–output relationship of Eq. (12) is exact for the case of stationary processes. Further, the expressions for the equivalent linear properties (ELPs) defined in Eq.(11) usually involve estimations of the response displacement and velocity variances determined by

$$E[x^2(t)] = \int_{-\infty}^{\infty} S_{xx}(t,\omega)\,d\omega \quad and \quad E[\dot{x}^2(t)] = \int_{-\infty}^{\infty} \omega^2 S_{xx}(t,\omega)\,d\omega. \quad (14)$$

It is noteworthy that for many nonlinear force-deformation laws the formulae assumption of Eq.(11) leads to closed-form expressions for the ELPs which considerably facilitate the application of statistical linearization. In the earthquake engineering field, the bilinear hysteretic force-deformation law, shown in Fig. 4(b), constitutes a commonly employed model to capture the hysteretic behavior of structural systems under seismic excitation (e.g. [14,20,21]). The governing equation of motion for a bilinear hysteretic oscillator can be expressed with the aid of an auxiliary state $z(t)$ (e.g. [15,22])

$$g_o\left(x(t),\dot{x}(t)\right) = \alpha x(t) + (1-\alpha)z(t), \quad (15)$$

with

$$\dot{\mathrm{z}}(\mathrm{t}) = \dot{x}(t)\left\{1 - \Phi\left(\dot{x}(t)\right)\Phi\left(z(t) - \mathrm{x}_y\right) - \Phi\left(-\dot{x}(t)\right)\Phi\left(-z(t) - \mathrm{x}_y\right)\right\}, \quad (16)$$

where $\Phi(\cdot)$ denotes the Heaviside step function, namely, $\Phi(m) = 1$ for $m \geq 0$, and $\Phi(m) = 0$ for $m < 0$, x_y is the yielding deformation and α is the post-yield to pre-yield stiffness ratio. It is readily conceived that marching towards $\alpha = 1.0$ reduces the nonlinear character of the system. Adopting the assumptions that the response of a viscously damped bilinear hysteretic SDOF oscillator is contained within a narrow frequency band and that the PDF of its response amplitude is a Rayleigh distribution, the following closed-form expressions for the ELPs can be determined (e.g. [23-25])

$$\zeta_{\mathrm{eq}}(t) = \zeta_{\mathrm{o}}\frac{\omega_o}{\omega_{eq}(t)} + \left(\frac{\omega_o}{\omega_{eq}(t)}\right)^2 \frac{1-\alpha}{\sqrt{\pi v(t)}} erfc(\frac{1}{\sqrt{v(t)}}), \quad (17)$$

and

$$\omega_{\mathrm{eq}}^2(t) = \omega_{\mathrm{o}}^2\left\{1 - \frac{8(1-\alpha)}{\pi}\int_1^{\infty}\left[\frac{1}{u^3} + \frac{1}{v(t)u}\right]\sqrt{u-1}\,exp(\frac{-u^2}{v(t)})du\right\}, \quad (18)$$

where $v(t) = 2E[x^2(t)]/\mathrm{x}_y^2$. It can be readily seen that Eqs.(12-14) and Eqs.(17-18) constitute a coupled nonlinear system of algebraic equations to be solved iteratively for the system ELPs determination. In this setting, a simple iterative while-loop is sufficient to simultaneously satisfy the system equations until convergence of the equivalent linear parameters is achieved within a pre-specified tolerance (e.g. [2,26]).

2.2.2 Forced vibrational system properties identification through an iterative mechanism

The proposed iterative mechanism which is applied on the top of the iterative statistical linearization process performs an efficient identification of the force-dependent vibrational system character. Specifically, the proposed iterative scheme identifies and follows the altering in vibrational system properties as the system is excited in the nonlinear range, and adjusts appropriately the characteristics of the strong part of the imposed seismic excitation. In this setting, it serves towards building an alternative scaling pattern for the determination of the scaled images of the IM which bears enhanced relation to its damaging potential by considering the effect of the induced nonlinearity as it is depicted in the values of the force-dependent vibrational system properties. Note in passing that the excitation definition is conformed with contemporary aseismic codes provisions and specifically those defined by Eurocode 8 (EC8). At this point, it is deemed appropriate to note that the choice of EC8 is not binding and that provisions defined by various aseismic codes can readily be considered.

At the core of the proposed iterative mechanism lies the notion of iteratively updating the nominal damping ratio ζ of the input damped acceleration spectrum by the forced vibrational equivalent damping property ζ_{eq} estimates, upon convergence. This is achieved by enforcing equality, within some allowance, between the stochastically equivalent damping coefficient and the damping ratio of the input UHS. Lastly, once convergence between $\zeta_{\mathrm{eq}}^{(k)}$ and $\zeta_{\mathrm{eq}}^{(k-1)}$ is achieved after k iterations, the forced vibrational system properties, $\omega_{\mathrm{eq}}^{(k_{end})}$ and $\zeta_{\mathrm{eq}}^{(k_{end})}$ are determined. Note in passing that the identification of the dynamic character of the system as it is depicted through the values of the forced vibrational system properties cannot be determined following classic nonlinear RHA. The appropriateness of the proposed scheme is assessed in the light of numerical results presented in Figs. (2a) to (2c) along with further elucidating remarks. The presented case-study concerns a quiescent base-excited nonlinear SDOF system whose elastoplastic behavior is governed by the hysteretic relationship shown in Eqs.(15-16) whereas the yielding displacement x_y is considered equal to 5 cm; the post-yield to pre-yield stiffness ratio α is assumed to be equal to 0.15.

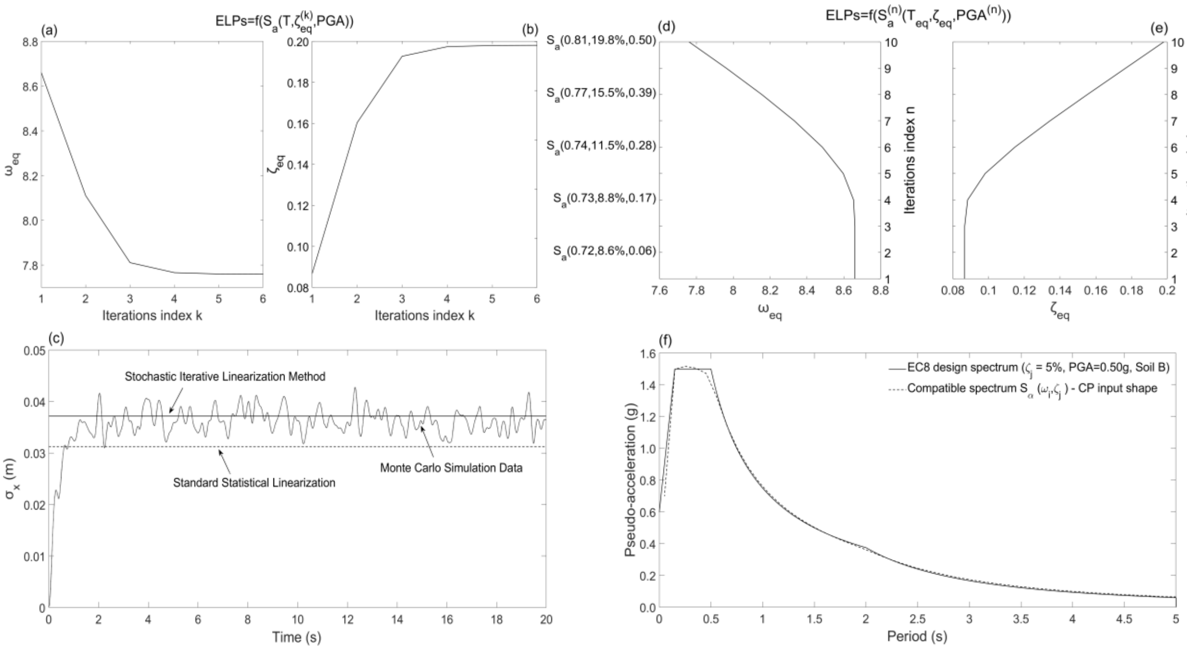

Figure 2. (a-b) Equivalent natural frequency and damping ratio coefficients provided by iterations on updating the nominal damping ratio of the input damped acceleration spectrum. (c) Comparison between standard statistical linearization technique with the proposed stochastic iterative linearization method in the light of MCS data (200 time-realizations) concerning the imposed excitation of Fig.2f. (d-e) Equivalent linear parameters provided as function of the introduced scalable ground motion intensity measure. (f) design spectrum defined by EC8; system properties $m = 20\ ton$, $k = 1.5\ MNm^{-1}$, and $c = 30\ kNsm^{-1}$.

From a numerical viewpoint, convergence of this iterative procedure (see Figs. 2a-b) is the natural outcome of solving at the current iteration a ‘‘system identification” problem to find ELPs for the SDOF oscillatory system, in which both the given input/excitation (i.e., response spectrum compatible spectrum) and the pre-specified output/response are found by relying on the ELPs identified in the previous iteration. It can be readily seen that convergence is achieved after six iterations for the SDOF bilinear system. The shown pertinent numerical results evidence that the proposed stochastic iterative linearization scheme as compared with the implementation of standard statistical linearization method provides with a significantly enhanced degree of accuracy in terms of response estimation while identifies reliably the stochastically equivalent forced vibrational system properties which are capable to track the system nonlinear character as well as tracing moving resonance phenomena. Note in passing that in cases where the degree of the exhibited nonlinearity is considerable, the contribution of the proposed iterative scheme becomes more evident; see Fig. 2c. Clearly, it ameliorates the well-reported in the literature accuracy of standard statistical linearization allowing for addressing cases of stronger nonlinear behavior. For discretization step $\Delta\omega = 0.1\text{rad/s}$, duration $T_s = 20s$, probability $p = 0.5$ and assumed parameters $\xi_g = 0.78$, $\omega_g = 10.78\ rad\ s^{-1}$, $\xi_f = 0.92$ and $\omega_f = 2.28\ rad\ s^{-1}$ the preselected Clough-Penzien (CP) spectral shape of the form

$$D(\omega_i) = \frac{(\omega_i/\omega_f)^4}{\left(1-(\omega_i/\omega_f)^2\right)^2 + 4\xi_f^2(\omega_i/\omega_f)^2} \frac{\omega_g^4 + 4\xi_g^2\omega_g^2\omega_i^2}{(\omega_g^2-\omega_i^2)^2 + 4\xi_g^2\omega_g^2\omega_i^2} \tag{19}$$

is employed. The parameters ω_g and ξ_g describe the filtering effects of the geological formations on the propagation of the seismic waves, while ω_f and ξ_f control the incorporated CP high-pass filter to suppress the low frequency content. The spectral moments in Eq. (3) are computed using the trapezoidal rule whereas the achieved level of compatibility between the power spectrum $G^{\zeta_j}(\omega)$ and the response spectrum S_a is assessed in Fig. 2f by comparing the given S_a of Eurocode 8 with the response spectrum computed by Eq. (1) (broken line).

2.2.3 Determination of a rigorous scalable ground motion intensity measure

An incremental mechanization analogous to the one used in the standard implementation of IDA is proposed and applied herein. However, the scalable IM has consciously been selected to be the damped spectral acceleration considering appropriately the force-dependent vibrational properties of the structure, meaning $S_a(\mathrm{T}_{\mathrm{eq}}^{(k_{end})}, \zeta_{\mathrm{eq}}^{(k_{end})}, PGA)$ where $\mathrm{T}_{eq} = 2\pi/\omega_{eq}$. In this setting, the scaled images of the IM bear enhanced relation to the associated damaging potential of the seismic excitation. For clarity reasons, the overall proposed framework is schematically depicted in the flowchart of Fig. 3.

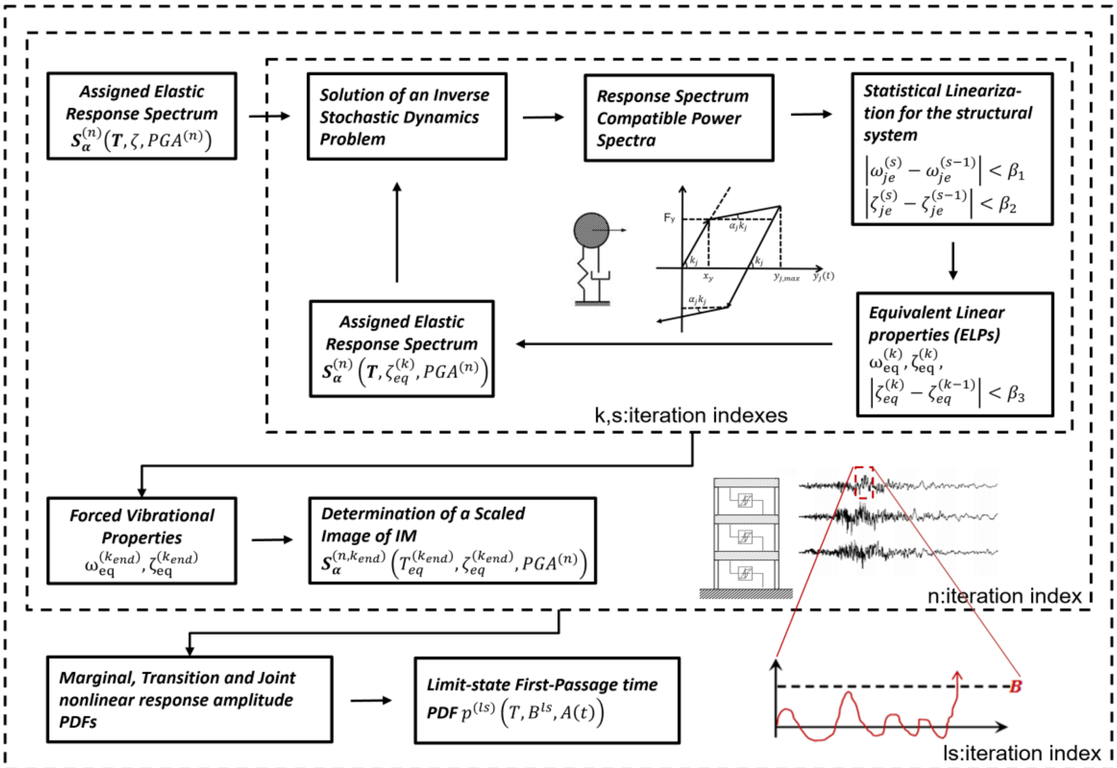

Figure 3. Flowchart of the proposed first-passage PDF-based stochastic IDA methodology.

Note in passing that the most widely adopted approach for the IM is to utilize the five percent damped spectral acceleration at the pre-yield first mode period of the structure $S_a(T_1, 5\%)$.

However, such an approach disregards significant information regarding the potential nonlinear effects which a scaled-up image of the input acceleration can have on a given structure. The pertinent numerical results shown in Figs. (2d) and (2e) evidence that the forced dependent vibrational properties corresponding to every scaled image of the IM are amenable to a clear physical interpretation. Actually, they appear to capture the inelastic response of the system depending on the excitation intensity (i.e. levels of IM) by taking on values in alignment with engineering intuition. Specifically, stronger nonlinear response due to marching towards higher excitation IM leads to heavier damped oscillators shifted towards lower frequencies.

2.3 *Marginal, transition and joint nonlinear system response amplitude PDFs*

The notion of the first-passage problem that is introduced herein, is related to the evaluation of the probability that the system response crosses a predetermined barrier level for the first time over a given time interval signaling that damages associated with a specific limit-state start to be suffered by the structure. In the herein study, the interest lies on the determination of the density function of the time moment when the system response reaches and exceeds a boundary (i.e. limit-state rule) for the first time given a specific level of the selected IM (e.g. [27]). In cases where the boundary is set relatively low, or the excitation level is considerably high, the barrier exceedance may reasonably be expected to occur at an early stage of the response process. On the course of demonstrating and better tracking potential barrier violation, a deterministic normalized time function of the kind $w(t) = Ctexp(-bt/2)$ with constants C and b, is assumed to modulate the power spectrum of the associated underlying stochastic process corresponding to the strong part of the ground motion input.

In the ensuing analysis, the assumption of lightly damped systems (i.e. $\zeta_{\mathrm{eq}}(t) \ll 1$) is made. In this regard, it can be argued that the oscillatory response $x(t)$ of the linearized system of Eq. (9) trails a pseudo-harmonic behavior described by the equation

$$x(t) = A(t)cos\,(\omega_{eq}(t)t + \varphi(t)), \tag{20}$$

where $\varphi(t)$ stands for the phase of the response. The response amplitude $A(t)$ is a slowly varying function with respect to time given as

$$A(t) = \sqrt{x^2(t) + \left(\frac{\dot{x}(t)}{\omega_{eq}(t)}\right)^2} \tag{21}$$

Furthermore, a combination of deterministic and stochastic averaging procedure leads to a first-order stochastic differential equation (SDE) governing the oscillatory response amplitude process $\mathrm{A}(t)$ given as

$$\dot{A}(t) = -\zeta_{eq}(t)\omega_{eq}(t)A(t) + \frac{\pi S_{f_o f_o}(\zeta_j, t, \omega_{eq}(t))}{2\omega_{eq}^2(\mathrm{t})\mathrm{A}(t)} + \frac{\sqrt{\pi S_{f_o f_o}(\zeta_j, t, \omega_{eq}(t))}}{\omega_{eq}(t)}\,\eta(t) \tag{22}$$

where $\eta(t)$ stands for a stationary, zero mean and delta correlated Gaussian white noise process of unit intensity and $S_{f_o f_o}(\zeta_j, t, \omega) = |w(t)|^2 G^{\zeta_j}(\omega)$. It is noted that the amplitude in Eq.(22) is decoupled from the phase $\varphi(t)$, thus it can be treated as an one-dimensional Markov process (e.g. [27-29]). The transition probability density function $p(A_l, t_l|A_{l-1}, t_{l-1})$ of the oscillatory response amplitude process is provided by addressing the appropriate Fokker-Planck equation which yields

$$p(A_l, t_l|A_{l-1}, t_{l-1}) = \frac{A_l}{c_{l-1,l}} exp\left(-\frac{{A_l}^2 + {A_{l-1}}^2 \exp\left(-2\zeta_{eq}(t_{l-1})\omega_{eq}(t_{l-1})\tau_l\right)}{2c_{l-1,l}}\right)\dots$$
$$I_0\left(\frac{A_l A_{l-1}\exp\left(-\zeta_{eq}(t_{l-1})\omega_{eq}(t_{l-1})\tau_l\right)}{c_{l-1,l}}\right), \qquad (23)$$

where $\tau_l = t_l - t_{l-1}$ is the transition time which is generated through a discretization of the time domain into intervals of the form $[t_{l-1}, t_l]$, $l = 1,2, \dots, N$, with $t_0 = 0$ and $t_N = T$. The I_0 represents the modified Bessel function of the first kind and of zero order. In the herein study the length of the time interval is selected according to the pattern $\tau_l = t_l - t_{l-1} = 0.5T_{eq}(t_{l-1})$. Next,

$$c_{l-1,l} = \frac{\pi}{\omega_{eq}^2(t_{l-1})}\exp\left(-2\zeta_{eq}(t_{l-1})\omega_{eq}(t_{l-1})t_l\right)\dots$$
$$\int_{t_{l-1}}^{t_l} \exp\left(2\zeta_{eq}(t_{l-1})\omega_{eq}(t_{l-1})\hat{t}\right) S_{f_o f_o}\left(\zeta_j, \hat{t}, \omega_{eq}(t_{l-1})\right) d\hat{t}. \qquad (24)$$

where the involved subscripts appear on the bounds of the integral. For the sake of brevity, the following notation is employed $c_l \triangleq c_{0,l}$ and $c_{l-1} \triangleq c_{0,l-1}$ representing the time-dependent variance of the response amplitude. Next, by introducing the parameter

$$r_l^2 = \frac{c_{l-1}}{c_l}\exp\left(-2\zeta_{eq}(t_{l-1})\omega_{eq}(t_{l-1})\tau_l\right), \qquad (25)$$

the transition PDF $p(A_l, t_l|A_{l-1}, t_{l-1})$ of the oscillatory response amplitude process reads

$$p(A_l, t_l|A_{l-1}, t_{l-1}) = \frac{A_l}{c_l(1-{r_l}^2)} exp\left(-\frac{{A_l}^2 c_{l-1} + {A_{l-1}}^2 c_l {r_l}^2}{2c_{l-1}c_l(1-{r_l}^2)}\right) I_0\left(\frac{A_{l-1}A_l r_l}{\sqrt{c_{l-1}c_l(1-{r_l}^2)}}\right). \qquad (26)$$

It has been shown that the probability function of the response amplitude process $A(t)$ follows a distribution of the Rayleigh type (e.g. [30])

$$p(A_{l-1}, t_{l-1}) = \frac{A_{l-1}}{c_{l-1}} exp\left(-\frac{{A_{l-1}}^2}{2c_{l-1}}\right), \qquad (27)$$

Relying on the Markovian assumption for the process $A(t)$ the joint oscillatory response amplitude PDF is provided as

$$p(A_{l-1}, t_{l-1}; A_l, t_l) = p(A_{l-1}, t_{l-1})\, p(A_l, t_l|A_{l-1}, t_{l-1}) \qquad (28)$$

Next, considering and manipulating Eqs.(26-28) yields the following expression for the joint response system amplitude PDF

$$p(A_{l-1}, t_{l-1}; A_l, t_l) = \frac{A_{l-1}A_l}{c_{l-1}c_l(1-{r_l}^2)} exp\left(-\frac{{A_l}^2 c_{l-1} + {A_{l-1}}^2 c_l}{2c_{l-1}c_l(1-{r_l}^2)}\right) I_0\left(\frac{A_{l-1}A_l r_l}{\sqrt{c_{l-1}c_l(1-{r_l}^2)}}\right) \qquad (29)$$

which is identified as an essential prerequisite in the process of defining the limit-state first-excursion probability provided in the following subsection.

2.4 Bilinear hysteretic system limit-state rule first-passage PDF determination

In the herein study, the employed model assumes that the structure enters a limit/damage state when the response amplitude $A(t)$ reaches for the first time a specific limit-state rule. In this regard, the non-excursion probability $P^{LS}(T, B^{LS}, A(t))$ is defined as the probability that the system response amplitude $A(t)$ does not exceed a prescribed limit-state barrier B^{LS} over the time interval $[t_0 = 0, T]$, namely $Prob[A(t) \leq B^{LS}]$ given that $A(t_0) \leq B^{LS}$. The first-excursion time is introduced herein as a random variable characterized by the corresponding PDF $p^{LS}\big(T, B^{LS}, A(t)\big)$ determined as

$$p^{LS}\big(T, B^{LS}, A(t)\big) = -\frac{d}{dT}\left(P^{LS}(T, B^{LS}, A(t))\right) \quad (30)$$

The selected time domain discretization scheme secures that the response amplitude $A(t)$ can be treated as constant over the time interval $[t_{l-1}, t_l]$, owing to its slowly varying character with respect to time [27]. Hence, the non-excursion probability $P^{LS}(T, B^{LS}, A(t))$ is assumed as well to be constant over $[t_{l-1}, t_l]$. In this setting, the non-excursion probability is defined as

$$P^{LS}\big(T = t_N, B^{LS}, A(t)\big) = \prod_{l=1}^{N} [1 - F_l^{LS}], \quad (31)$$

where F_l^{LS} is defined as the probability that the system response amplitude $A(t)$ will cross the limit-state barrier B^{LS} in the time-interval $[t_{l-1}, t_l]$, given that no exceedance has been occurred prior to time t_{l-1}. Further, invoking the standard definition of conditional probability (e.g. [31]) yields

$$F_l^{LS} = \frac{Prob[A_l \geq B^{LS} \cap A_{l-1} < B^{LS}]}{Prob[A_{l-1} < B^{LS}]} = \frac{D_{l-1,l}^{LS}}{Q_{l-1}^{LS}}, \quad (32)$$

where ∩ denotes the intersection symbol. Considering Eq. (27) the denominator of Eq.(32) can be determined analytically in a direct manner as

$$Q_{l-1}^{LS} = \int_0^{B^{LS}} p(A_{l-1}, t_{l-1})\, dA_{l-1} = 1 - exp\left(-\frac{(B^{LS})^2}{2c_{l-1}}\right), \quad (33)$$

whereas the nominator is defined as

$$D_{l-1,l}^{LS} = \int_{B^{LS}}^{\infty} dA_l \int_0^{B^{LS}} p(A_{l-1}, t_{l-1}; A_l, t_l) dA_{l-1}. \quad (34)$$

Next, by expanding the Bessel function $I_0(\psi)$ (e.g. [32]) appearing in Eq.(29) in the form

$$I_0(\psi) = \sum_{\theta=0}^{\infty} \frac{(\psi/2)^{2\theta}}{\theta!\, \Gamma(\theta+1)}, \quad (35)$$

and manipulating Eq.(34) an analytical treatment for the involved integrals is possible yielding

$$D_{l-1,l}^{LS} = \Delta_0^{LS} + \sum_{\gamma=1}^{M} \Delta_\gamma^{LS}, \quad (36)$$

where

$$\Delta_0^{LS} = exp\left(-\frac{(B^{LS})^2}{2c_l(1-r_l^{\ 2})}\right)\left(1-exp\left(-\frac{(B^{LS})^2}{2c_{l-1}(1-r_l^{\ 2})}\right)\right)(1-r_l^{\ 2}), \tag{37}$$

and

$$\Delta_\gamma^{LS} = L_\gamma \frac{r_l^{\ 2\gamma}}{(c_{l-1}c_l)^{\gamma+1}(1-r_l^{\ 2})^{2\gamma+1}\prod_{\gamma=1}^{M}(2\gamma)^2}, \tag{38}$$

with

$$L_\gamma = 4^\gamma c_{l-1}{}^{\gamma+1} c_l (1-r_l^{\ 2})^{\gamma+2} \times \left(\Gamma\left[1+\gamma, \frac{(B^{LS})^2}{2c_{l-1}(1-r_l^{\ 2})}\right] - \Gamma[1+\gamma,0]\right)\ldots$$
$$\left(-(c_l(1-r_l^{\ 2}))^\gamma \Gamma[1+\gamma] + \left(\frac{(B^{LS})^2}{c_l(1-r_l^{\ 2})}\right)^{-\gamma}\left(\Gamma[1+\gamma]-\Gamma\left[1+\gamma,\frac{(B^{LS})^2}{2c_l(1-r_l^{\ 2})}\right]\right)B^{2\gamma}\right). \tag{39}$$

The terms $\Gamma[s]$ and $\Gamma[\eta, s]$ appearing at Eq. (39) represent the Gamma function and the upper incomplete Gamma function, respectively. Notably, the herein proposed technique manages to keep the associated computational cost at a minimum level.

2.5 Definition of limit-states for efficient system first-passage time PDF estimates

There is a considerable body of studies in the relevant literature, where the limit-state rule is defined in terms of the overall system inelastic deformation or the maximum inter-story drift (e.g. [25,33]) In the herein study, it is deemed appropriate to study the problem from a first-passage perspective (e.g. [34,35]) resorting to the selection of the limit-state first-excursion time as the EDP. Note in passing that the selected scalable IM of the damped spectral acceleration considering the level of the exhibited nonlinearity, as depicted in the force-dependent vibrational properties bears a direct relation to its damaging potential. Further, due to the alternative nature of the selected EDP $p^{LS}(T, B^{LS}, A(t))$ the proposed stochastic IDA methodology provides the functional relationship between the IMs and the EDPs in conjunction with LSs. It should be recalled that the standard IDA technique is concerned with the estimation of the relation between IMs and EDPs. The first-excursion model requires the definition of a vector of limit-state rules; a possible mapping between performance requirements and system limit-states, expressed in terms of inter-story drift, for a typical building structure is provided in Table 1 (e.g. [25]).

Table 1. Performance requirements and limit states

Limit States	Limit-state barrier B^{LS}
Impaired function	3.0×10^{-2}
Life safety	5.0×10^{-2}
Onset of collapse	9.0×10^{-2}

2.6 Discussion

A discussion on a number of notable attributes which concerns advantages, limitations as well as potential practical applications of the proposed framework is presented herein.

Comparing to the state of the art schemes available in the literature, the proposed first-passage PDF-based stochastic IDA methodology exhibits a number of noteworthy and intriguing attributes such as: (i) it accounts for nonlinear structural systems following a behavior of the hysteretic kind; (ii) the ground motion is modeled in the form of a vector of code-compliant stochastic processes rather than a suite of scaled earthquake records (e.g. Multi-record IDA). In this setting, the challenge of selecting and scaling ground motion records is conveniently avoided; note in passing that the issue still remains highly controversial in the relevant literature (e.g. [8,9]); (iii) the proposed methodology reduces drastically the induced bias stemming from the selection of a limited number of seismic motion records, typically around 7; (iv) the commonly adopted five percent damped spectral acceleration at the pre-yield first mode period of the structure $S_a(T_1, 5\%)$ is abandoned in favor of a more comprehensive IM which considers thoroughly information regarding potential nonlinear effects that a scaled-up excitation image can induce into the structure. In this setting, the proposed scaling process treats in a consistent manner the need for relating the scaled excitation with its damaging potential on the structure. This is achieved by considering the induced nonlinearity as it is depicted in the values of the forced vibrational system properties in the determination of the scaled images; (v) at the first-stage of the proposed framework lies an efficient stochastic iterative linearization methodology which exhibits enhanced levels of accuracy in terms of response determination as compared with the standard implementation of statistical linearization. This attribute renders it appropriate for handling nonlinear behaviors of the heavier kind, providing reliable response estimates as well as robust estimations for the forced equivalent linear parameters based on the degree of the exhibited nonlinearity, thus offering a solid basis for interpreting the dynamic character of the system; (vi) it determines higher order statistics of the selected EDP (i.e. PDF of the first-passage time) rather than simple estimates only of the mean and the standard deviation currently being the norm in the literature. Clearly, performing IDA within a probabilistic framework, (i.e. considering the seismic excitation modeled as a stochastic process) and depending on the required response statistical quantities (i.e. mean, standard deviation, or the PDF), hundreds or even thousands of IDA curves are typically required within a MCS context for a reliable statistical description of the EDP. Obviously, performing a standard brute force implementation of the IDA methodology for a MCS based determination of the EDP statistics can be a computationally prohibitive task; (vii) the well-observed back-and-forth twisting patterns of IDA curves are associated with vagueness regarding the satisfaction of a rule which signals reaching a limit state (e.g. [6]). In an attempt to address timing rather than just intensity peculiarity, the herein study introduces the limit-state first-excursion time as an appropriate EDP; (viii) it is considerably less computationally demanding compared to nonlinear RHA for compatible ground motion records; (ix) it provides a threefold functional relationship between the IMs and the EDPs in conjunction with LSs. Note in passing that the standard IDA technique which is concerned with the estimation of the relation between the first two variables necessitates a careful handling of the subsequent coupling with the LSs for interpreting potential conformity with a particular performance level.

Pertinent remarks should be provided regarding the expected level of accuracy since the developed method for the sake of efficiency encompasses a number of techniques which bear plausible limitations. The proposed stochastic iterative linearization methodology demonstrates an enhanced degree of accuracy as compared with the implementation of standard statistical linearization method for applications in the structural engineering field. Specifically, in cases where the degree of the exhibited nonlinearity does not belong necessarily into the mild kind, the proposed scheme demonstrates considerable improvement. However, the proposed methodology would not provide with sufficiently accurate estimates for cases of particularly low-performing structures. Under such conditions, the combination of deterministic with stochastic averaging treatment proposed for the equivalent linearized oscillator in favor of substantial computational efficiency may lessen the achieved degree of accuracy; for cases of highly nonlinear behavior the assumption of the equivalent light damping (i.e. $\zeta_{\mathrm{eq}}(t) \ll 1$) may be violated. Finally, no restrictions are imposed on the excitation, with the only exception being the Gaussian assumption.

The first-excursion definition has clearly a particularly critical meaning for the most severe damage/limit state, where potential exceedance may lead to total collapse. This attribute enriched with timing information may lie closer to the principle idea of the Dynamic Pushover Analysis (DPO) which was conceived as a way to estimate a proxy for the global collapse of structure. However, in agreement with the current IDA the proposed first-passage PDF-based SIDA methodology retains the advantageous attribute of studying multiple limit-states (e.g. that of life safety which represent one of the most commonly employed level of threats for seismic design applications).

3 ILLUSTRATIVE APPLICATION

In this section the proposed first-passage PDF-based stochastic IDA methodology is numerically exemplified by considering a yielding frame structure subject to stochastic seismic excitation in alignment with specifications prescribed by contemporary aseismic codes and specifically that of EC8. The achieved degree of accuracy is assessed by comparison with pertinent results derived from nonlinear RHA for a large ensemble of acceleration time-realizations compatible with the code-compliant underlying stochastic processes.

3.1 Building structure with nonlinearity of the hysteretic kind

The one-story inelastic shear frame shown in Fig. 4a is considered to illustrate the proposed approach. The vibration of the structure, in the plane shown, is induced as a result of the imposed seismic action $f(t)$. The lumped mass m, the stiffness and damping coefficients k and c, respectively, are provided as $m = 20\ ton$, $k = 1.5\ MNm^{-1}$, and $c = 30\ kNsm^{-1}$. The elastoplastic behavior of the shear frame is governed by a hysteretic relationship between the resisting story shearing force and the corresponding drift shown in Eqs.(15-16) whereas the yielding displacement x_y is taken equal to 5 cm. Lastly, the post-yield to pre-yield stiffness ratio α is assumed to be equal to 0.15 implying a strong nonlinear character.

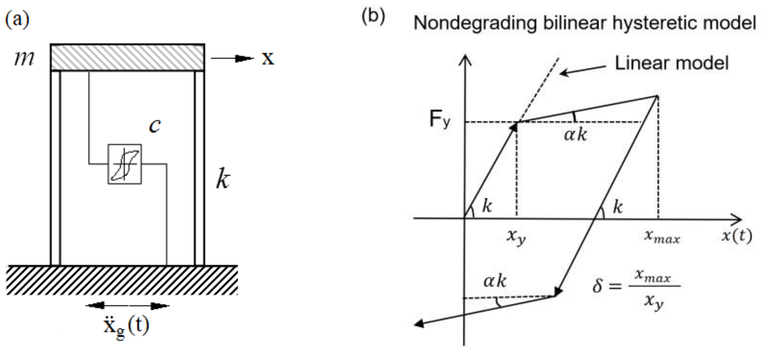

Fig. 4. (a) One-story inelastic building structure, (b) the governing bilinear hysteretic restoring force-deformation law and definition of ductility ratio δ

3.2 Limit-state first-passage time IDA density functions

Following the efficient nonlinear first-passage stochastic IDA methodology delineated in sections 2.1-2.5, higher order statistics (i.e. PDF) of the response limit-state first-excursion time are efficiently determined. The achieved level of accuracy is presented in Figs. 5a-f by comparing proposed methodology results with pertinent MCS data involving a large ensemble of $5{,}000$ acceleration time-histories generated compatibly with the imposed response spectrum specifications of the introduced scalable IM (e.g. [36]). Next, the nonlinear differential equation of motion in Eq.(7) is numerically integrated via a standard fourth order Runge-Kutta scheme, and system first-passage time response statistics are obtained based on the ensemble of the response realizations.

The adopted thresholds for checking the convergence are set to $\beta_1 = \beta_2 = \beta_3 = 10^{-4}$ whereas a deterministic normalized time function $w(t)$ with constant values $C = 0.1776$ and $b = 0.10$ is assumed ensuring a progressive escalation of excitation intensity up to the imminent level. Following the implementation of the proposed framework as shown in Fig. 3, the limit state first-passage IDA PDFs are efficiently determined and compared with the corresponding MCS data. Specifically, in Figs. 5a and 5b the first-passage time IDA PDFs corresponding to the limit state of *Impaired function* are provided. Pertinent results for the totality of the considered limit states can be found in the subsequent Figs. 5c-f.

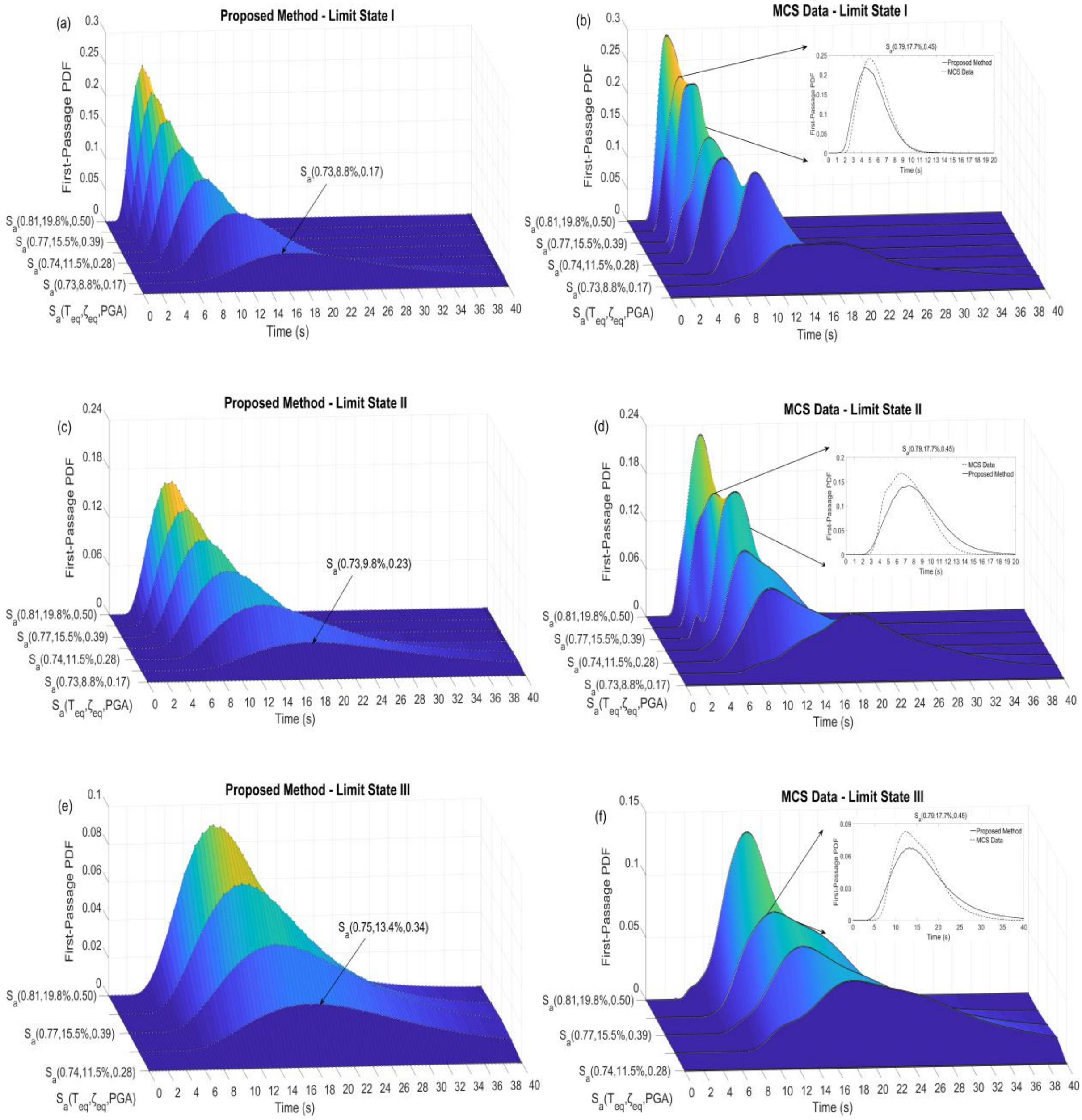

Figure 5. First-passage time IDA PDF estimates of the elastoplastic shear frame shown in Fig. 4a for the limit state *Impaired function* (a) through the proposed methodology (b) through MCS, for the limit state *Life safety* (c) through the proposed methodology (d) through MCS, and, for the limit state *Onset of collapse* (e) through the proposed methodology (f) through MCS.

The indicated lower bound appearing in each and every one of the considered limit-states is associated with the critical value in IM terms which signals the entrance of a structure into a specific limit-state; no barrier violation has been observed for scaled down images of the IM. Notably, the limit state first-passage time IDA PDFs are determined at a minimum computational cost, harnessing the potential of the developed stochastic dynamics methodology outlined in section 2. Evidently, comparisons with MCS data reveal a satisfactory degree of accuracy. This observation renders the proposed technique appropriate for related performance-based engineering applications. Further, the forced equivalent linear parameters based on the degree of the exhibiting nonlinearity provide a solid basis for interpreting the underlying dynamic character of the system. The complete statistical description of the EDP is provided by the PDFs, whereas the derivation of other EDP related statistical quantities (e.g. mean, mode etc) can be readily identified as a straightforward task. Remarkably, a potential consideration of the modes (most probable value) of the herein employed EDP can lead to a coveted one-to-one mapping for the functional relationship between the IMs and the EDPs in conjunction with the LSs avoiding pertinent peculiarities related to non-monotonic behavior, commonly found in standard IDA curves. Note that the proposed method leads to substantial reduction of the computational effort as compared with nonlinear RHA within a MCS framework. In this setting, to provide with an indicative order of magnitude for the computational cost involved, utilizing a laptop computer with standard configurations, the proposed technique needs 10-12 min for determining the first-passage time IDA PDFs for a single limit-state, whereas the MCS based estimation involving 5,000 time-histories requires 11-12 h for estimating reliably higher order statistics (e.g. the PDF).

The generation of a single standard IDA curve can last from 30 seconds to 1 hour, while in the case of a multi-record IDA where a number of curves (usually ≤ 20) is generated the processing time can be significantly increased. Note, however, that a statistical analysis based on this number of records could provide with reliable estimates only of the mean and presumably the standard deviation of the chosen EDP. A significantly higher number of records would be needed to estimate reliably higher order statistics (or the PDF). Clearly, there is a potential of the proposed stochastic IDA methodology for addressing high-dimensional MDOF systems, a fact which makes the approach highly recommended for computational demanding fully probabilistic PBEE analysis frameworks. The low computational cost attribute hopefully qualifies the herein proposed approach as a potent analysis tool at least for preliminary stochastic response analysis of yielding structures. It is worth mentioning that the seismic demands are imposed in alignment with the aseismic codes dictated by the EC8, however, the proposed approach can readily be modified to handle specifications prescribed by any contemporary code of practice dealing with various kinds of hazards (e.g. ocean-waves, winds, hurricanes, tsunamis etc).

4 CONCLUDING REMARKS

This paper proposes an efficient stochastic incremental dynamics methodology considering first-excursion probability for hysteretic structural systems subject to stochastic seismic excitations in alignment with contemporary aseismic codes provisions. Initially, an efficient stochastic iterative linearization methodology is devised achieving convergence of the equivalent damping ratios with the damping premises of the excitation response spectrum leading to a coherent determination of a robust scalable intensity measure (IM) which bears direct relation to its damaging potential. Subsequently, utilizing the stochastically derived time-varying forced vibrational system properties for each scaled IM level in conjunction with a combination of deterministic and stochastic averaging treatment the first-passage time probability density function (PDF) is efficiently determined for any considered limit-state rule. Lastly, an incremental mechanization analogous to the one used in normal incremental dynamic analysis is proposed to ensure the necessary compatibility for pertinent applications in structural engineering field.

The proposed stochastic dynamics methodology provides with reliable higher order statistics of the selected engineering demand parameter (EDP) rather than simple estimates only of the mean and the standard deviation currently being the norm in the literature. The selected EDP of the first-excursion time which signals the entrance of a structure into a specific limit-state constitutes an excellent response related variable with twofold meaning; it performs structural behavior monitoring considering intensity as well as timing information whereas it is naturally coupled with limit-state requirements. Note in passing that the standard IDA technique necessitates a separate handling of the coupling between the IDA curves (i.e. functional relation between IMs and EDPs) with the limit-state rules (LSs) which involves interpretation regarding the potential conformity or non-conformity of structural behavior with a particular performance level. Further, the associated low computational cost renders the proposed methodology particularly useful for related performance-based engineering applications. The concepts involved have been numerically illustrated using a bilinear hysteretic frame structure exposed to ground motion modeled in accordance with contemporary aseismic code provisions. Lastly, nonlinear response time-history analysis involving a large ensemble of non-stationary acceleration time-histories has been conducted to assess the accuracy of the proposed framework in a Monte Carlo-based context.

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REFERENCES

[1] Kong F., Spanos P. D., Li J., Kougioumtzoglou I. A., Response evolutionary power spectrum determination of chain-like MDOF nonlinear structural systems via harmonic wavelets, *International Journal of Non-Linear Mechanics, vol. 66: 3-17, 2014*.

[2] Mitseas I. P., Kougioumtzoglou I. A., Giaralis A., Beer M. A novel stochastic linearization framework for seismic demand estimation of hysteretic MDOF systems subject to linear response spectra, *Structural Safety,* vol. 72: 84-98, *2018*.

[3] Der Kiureghian A. Non-ergodicity and PEER's framework formula. *Earthquake Engineering and Structural Dynamics, vol. 34(13), p. 1643–52, 2005.*

[4] Tubaldi E., Barbato M., Dall' Asta A., Performance-based seismic risk assessment for buildings equipped with linear and nonlinear viscous dampers, *Engineering Structures 78, 90-99, 2014.*

[5] Mitseas, I.P., Kougioumtzoglou, I.A., Beer, M. An approximate stochastic dynamics approach for nonlinear structural system performance-based multi-objective optimum design. *Structural Safety,* vol. 60, p. *67-76, 2016.*

[6] Vamvatsikos D., Cornell C.A. Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics 2002; 31(3): 491–514*.

[7] Vamvatsikos D., Fragiadakis M., Incremental dynamic analysis for estimating seismic performance sensitivity and uncertainty, *Earthquake Engineering and Structural Dynamics, 2010; 39 (2), 141-163.*

[8] Grigoriu M., To Scale or Not to Scale Seismic Ground-Acceleration Records, *Journal of Engineering Mechanics;137:284-293, 2011.*

[9] Grigoriu M. Do seismic intensity measures (IMs) measure up?, *Probabilistic Engineering Mechanics; 46:80–93, 2016.*

[10] CEN. Eurocode 8: Design of Structures for Earthquake Resistance - Part 1: General Rules, Seismic Actions and Rules for Buildings. EN 1998-1: 2003, Brussels, 2004.

[11] Vanmarcke EH. Properties of spectral moments with applications to random vibration. *Journal of Engineering Mechanics 1972; 98(2):425–46*.

[12] Vanmarcke EH. *Structural response to earthquakes. In: Lominitz C, Rosenblueth E, editors. Seismic risk and engineering decisions*. New York: Elsevier; 1976. p. 287–337.

[13] Cacciola P., Colajanni P., Muscolino G., Combination of modal responses consistent with seismic input representation. *Journal of Structural Engineering ASCE 2004; 130: 47–55*.

[14] Mitseas I. P., Beer M., Modal decomposition method for response spectrum based analysis of nonlinear and non-classically damped systems, *Mechanical Systems and Signal Processing, vol. 131: 469–485, 2019.*

[15] Roberts J.B., Spanos P.D., *Random vibration and statistical linearization*, New York: Dover Publications, *2003*.

[16] Priestley MB. Power spectral analysis of non-stationary random processes. *Journal of Sound and Vibration 1967; 6:86–97*.

[17] Dahlhaus R. Fitting time series models to non-stationary processes. *The Annals of Statistics 1997; 25:1–37*.

[18] Hammond JK. Evolutionary spectra in random vibration. *Journal of the Royal Statistical Society 1973; 35:167–88*.

[19] Kougioumtzoglou I. A., Spanos P. D., Nonlinear MDOF system stochastic response determination via a dimension reduction approach, *Computers and Structures; 126: 135-148, 2013.*

[20] Ramirez OM, Constantinou MC, Gomez JD, Whittaker AS, Chrysostomou CZ. Evaluation of simplified methods of analysis of yielding structures with damping systems. *Earthquake Spectra 2002; 18:501–30*.

[21] Chopra A.K., Chintanapakdee C. Inelastic deformation ratios for design and evaluation of structures: single-degree-of-freedom bilinear systems. *Journal of Structural Engineering ASCE 2004;130:1309–19*.

[22] Suzuki Y, Minai R. Application of stochastic differential equations to seismic reliability analysis of hysteretic structures. *Probabilistic Engineering Mechanics 1988; 3: 43–52*.

[23] Caughey T.K. Random excitation of a system with bilinear hysteresis. *Journal of Applied Mechanics ASME 1960; 27:649–52.*

[24] Iwan W. D, and Lutes L. D., Response of the Bilinear Hysteretic System to Stationary Random Excitation*, Journal of the Acoustical Society of America 1968, 43 (3). pp. 545-552.*

[25] Mitseas I. P., Beer M., Fragility analysis of nonproportionally damped inelastic MDOF structural systems exposed to stochastic seismic excitation, *Computers and Structures, vol. 226, doi: 10.1016/j.compstruc.2019.106129*, 2020.

[26] Kougioumtzoglou I.A, Fragkoulis V.C., Pantelous A.A., Pirrotta A., Random vibration of linear and nonlinear structural systems with singular matrices: A frequency domain approach, *Journal of Sound and Vibration 2017,* Vol. 404, 84-101.

[27] Mitseas, I.P., Kougioumtzoglou, I.A., Spanos P.D., Beer, M. Nonlinear MDOF system Survival Probability Determination Subject to Evolutionary Stochastic Excitation. *Journal of Mechanical Engineering; 62 7-8, 440-451, 2016.*

[28] Spanos, P.D, Kougioumtzoglou, I.A. (2014). Survival probability determination of nonlinear oscillators subject to evolutionary stochastic excitation. *Journal of Applied Mechanics, vol. 81, no. 5, 051016, DOI:10.1115/1.4026182*.

[29] Mitseas I. P., Kougioumtzoglou I. A., Spanos P. D., Beer M., Reliability assessment of nonlinear MDOF systems subject to evolutionary stochastic excitation, *Proceedings of the 7th International Conference on Computational Stochastic Mechanics (CSM 7), Santorini, Greece, 15-18 June, 2014, doi: 10.3850/978-981-09-5348-5_042*.

[30] Spanos P. D., Lutes L. D, Probability of response to evolutionary process, J. *Eng Mech. Div. Am. Soc. Civil Eng., 106, 213-224, 1980.*

[31] Ang AH-S, Tang WH. Probability concepts in engineering. 2nd ed. Wiley; 2007.

[32] Papoulis, A., *Probabilities, Random Variables, and Stochastic Processes. McGraw-Hill,* New York, 1965.

[33] Ellingwood BR., Earthquake risk assessment of building structures, *Reliability Engineering and System Safety;74: 251–62, 2001.*

[34] Jensen H.A., Mayorga F., Valdebenito M.A., On the reliability of structures equipped with a class of friction-based devices under stochastic excitation, *Computer Methods in Applied Mechanics and Engineering*, *Vol. 364, doi.org/10.1016/j.cma.2020.112965, 2020.*

[35] Au S.K., Beck J.L., First excursion probabilities for linear systems by very efficient importance sampling, *Probabilistic Engineering Mechanics Vol. 16, 193-207, 2001*.

[36] Shinozuka M., Deodatis G., Simulation of stochastic processes by spectral representation, *Applied Mechanics Reviews 1991, vol. 44, no. 4, pp. 191-204*.